2501. (a) The possibility space is a list of six pairs. List them, and pick out the relevant probabilities.
    (b) Restrict the possibility space.

2502. Differentiate using the chain rule, and find the equation of the tangent. Set $y = 0$.

2503. Expand and simplify top and bottom, looking to cancel a factor of k. Once there are no factors of k on the bottom, you can safely take the limit.

2504. You can find the equation of the tangent, or, as is probably easier, drop perpendiculars to the axes from the point $(\cos\theta, \sin\theta)$, and then use similar triangles.

2505. Proceed algebraically, or else consider the other three curves as reflections of $y = e^x$.

2506. Solve each of the quadratic inequalities, and find the intersection of the solution sets.

2507. Use the quotient rule to differentiate, then set the numerator to zero to find stationary points.

2508. This requires a careful sketch, even a careful plot. A “convex” quadrilateral means that none of the interior angles are reflex.

2509. Differentiate x^2y^2 by the product rule and implicit differentiation. Then substitute and simplify.

2510. Differentiate k^x from first principles, and then take out a factor of k^x. Substitute in the result given.

2511. (a) Consider the standard conversion to harmonic form $R\sin(t + \alpha)$. But you don’t need to find the phase α, only the amplitude R. This is the Pythagorean sum of the amplitudes of the individual sinusoids.
    (b) Consider the domain $[0, 2\pi)$ as the possibility space. Again, you don’t need to consider the phase shift (i.e. translation in the t direction) associated with α. Consider $T = R\sin t$.

2512. Use the product rule, quoting the derivatives of $\sec\theta$ and $\operatorname{cosec}\theta$.

2513. The distribution $B(2n, 1/2)$ is symmetrical. So, you need only calculate the probability of getting “n heads, n tails”, subtract this from 1, and then divide by two.

2514. (a) Differentiate, substitute into the DE, and then equate coefficients.
    (b) Continue to find b, then use the point $(2, 0)$ to find c.

2515. (a) Draw a force diagram and resolve vertically.
    (b) Compare the situation to freefall: are there vertical forces on the cone other than gravity?

2516. In each case, sketch the boundary equation. Each equation should produce two straight lines.

2517. Carry out the integrals, including a single constant $+c$. Then rearrange using laws of logarithms. At the end, exponentiate.

2518. Use a double-angle formula to write $\sin^2 x$ in terms of $\cos 2x$. Simplify, and consider the result as $y = \cos x$ transformed in various ways.

2519. (a) Use $Z \sim N(0, 1)$.
    (b) Use $\bar{Z} \sim N(0, 1/5)$.
    (c) No calculation is needed: use symmetry.

2520. To return to point A, the ant must circumnavigate one of the faces.

2521. Find the intersection of the lines by equating the x and y components and solving simultaneously. To do this you need to give the two instances of t different names: rename one s. Then substitute this point into the LHS of the ellipse equation, and compare the value to 3.

2522. Set $y = x - 1$, and rewrite as $x = y + 1$.

2523. This can be easily shown in terms of *energy*, which is beyond the syllabus. Instead, set the angle of projection as θ above the horizontal, then find the horizontal and vertical speeds at landing. Find the overall speed with Pythagoras.

2524. Consider the coefficients of x^2.

2525. $\mathbb{R} \setminus \mathbb{Z}$ means “the set $\mathbb{R}$, with any elements of the set $\mathbb{Z}$ removed from it.”

2526. Set up the standard formula for differentiation from first principles. Then use the usual technique for inlaid fractions: multiply top and bottom of the big fraction by the denominators of the little fractions.

2527. (a) Replace y.
    (b) Switch x and y.

2528. Use the third Pythagorean trig identity to get an expression for $\cot\frac{3\pi}{8}$. Then reciprocate.

2529. Find expressions for the sums Σx of each data set. Add these, then divide by the total number of data.

2530. Complete the square to find the centres and radii of the circles. Then compare the sum of the radii to the distance between the centres.

2531. Begin by assuming $a \leq b$, so $\min(a, b) = a$. Start with $x + y > k$, multiply by a, and go from there. Then generalise the proof to the case $b \leq a$.

2532. Consider the force exerted on the exhaust, either assuming each particle of exhaust takes 1 second to accelerate to 400 ms^{-1}, or by letting each particle of exhaust take a generic t seconds to accelerate. The answer is the same either way. Then use NIII to analyse the engine.

2533. In a possibility space consisting of four equally likely outcomes, consider the events $\{1, 2\}$, $\{3, 4\}$, $\{1, 3\}$.

2534. Set up triangles to the centre, splitting polygon and circles into n sectors.

2535. This is a quadratic in x: use the formula.

2536. Write z in terms of x, including a $\pm$. Then integrate by the reverse chain rule. Note that the integral is with respect to x, not z.

2537. Set up a 6×6 possibility space, and restrict it.

2538. (a) For differentiation, the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are constants. Use Pythagoras to find magnitudes. Also show that $m_1 m_2 = -1$.
(b) Differentiate again.
(c) Show that the second derivative is the negative of the original phase function.

2539. You could differentiate, but it is easier to consider the overall shapes of the curves.

2540. (a) Find the distances between coordinates $(0, 0)$, $(1/4, 1/16)$, etc. by Pythagoras, and sum these.
(b) Consider the fact that a straight line is the shortest path between two points.

2541. Substitute the second equation into the first, and use log rules to simplify and solve.

2542. Differentiate, and find the gradient of the tangent at point $x = a$. Then use the fact that the tan function calculates the gradient.

2543. Only one of these holds. Consider examples in which $n = 2$.

2544. (a) An undefined gradient means (if the curve is defined, as here) a tangent parallel to the y axis.
(b) Find the first and second derivatives.
(c) Consider the domain of the curve, then use parts (a) and (b), together with the behaviour as $x \to \infty$.

2545. Multiply up by the denominator to get a cubic equation. Spot a root and factorise.

2546. Split the rhombus into four right-angled triangles with sides

$$\left(\tfrac{1}{2}a, \tfrac{1}{2}b, \tfrac{1}{2}\sqrt{a^2 + b^2}\right).$$

Then use similar triangles.

2547. Use the standard technique for inlaid fractions: multiply top and bottom of the big fraction by the denominator(s) of the small fraction(s).

2548. Consider the equivalent question: tangents to sin, cos, tan which are parallel to the x axis.

2549. Factorise the quadratic as $(ax + b)(cx + d) = 0$, and use the factor theorem.

2550. Place A_1 and A_2, then place someone next to A_1 and call them B_1; you can do so without loss of generality. Then consider B_2, and so on.

2551. Sketch the graph $y = x^5 - x^4$ over the domain $(0, 1)$.

2552. Multiply up by the denominator of the LHS, and equate coefficients.

2553. (a) Write the area of the trapezium formed by A's graph in terms of the acceleration a, and equate it to 100.
(b) Write the area under B's graph in terms of the maximum speed, and equate it to 96.

2554. (a) Use log rules to simplify, then exponentiate.
(b) Reverse the process.

2555. The problem is the null hypothesis: it must be a firm enough assumption to work with.

2556. Give the cuboid dimensions $a \times b \times c$, and set up three equations with Pythagoras. These are linear simultaneous equations in a^2, b^2, c^2. Add two of the equations and subtract the third to isolate one of the variables.

2557. Differentiate implicitly for $\frac{dy}{dx}$, using the product and chain rules.

2558. Rearrange each to make its trig ratio the subject. Then square the equations and add them.

2559. (a) Draw a clear force diagram and take moments about the foot of the plank.
(b) Use the fact that the ladder is on the point of slipping, i.e. $F_{\max} = \mu R$.
(c) Eliminate R from the equations in (b).

2560. Express "the x axis" algebraically, and look for multiple roots.

2561. Use the multiplicity of the roots to express the curve in factorised form, using a and p. Explain why $a > 0$. Then multiply out to express b, c, d in terms of a and p. This will give you the signs of the coefficients.

2562. Find $u_1, u_2, u_3, u_4, ...$. You'll soon see the pattern.

2563. For each fraction, express the trigonometric ratios in terms of sin and cos. Then multiply up by the denominators of the inlaid fractions.

2564. (a) Use $F = ma$,
(b) Differentiate the acceleration, and set $\frac{da}{dt} = 0$.

2565. Note that each of the equations has constant term $+k$. This makes the question much easier than it would have been for any other constant term. If you're not sure how to proceed, solve the problem with $k = 0$ first.

2566. Assume, for a contradiction, that $\bar{x} > 1$. Then manipulate $S_{xx} = \sum x^2 - n\bar{x}^2 \geq 0$ to show that $\sum x^2$ and $\sum x$ cannot be the same.

2567. Prove this by direct argument. Use the cyclic quadrilateral theorem, and allied angles.

2568. Integrate twice with respect to x, and rearrange.

2569. Both squares are moving at constant velocity, and the second one has constant y position. So, find the times at which the vertices of the first square have a feasible y coordinate. For each of these, check the x coordinates.

2570. Consider the case $\mu_1 \neq \mu_2$.

2571. Solve simultaneously by elimination, to get an equation with only two unknown vectors.

2572. (a) Find the discriminant, simplify and factorise.
(b) Continue, using the quadratic formula, and show that both roots (for x^2) are negative.

2573. The given point of intersection allows you to use the factor theorem. Factorise and solve a quadratic to find the other two roots.

2574. (a) Set the first derivatives equal to each other.
(b) Set the moduli of the first derivatives equal to each other.

2575. You could argue via symmetry. But the integral can be inspected immediately, so its easier just to quote the standard result.

2576. Draw a clear sketch, which shows graphically the solution set of the inequality. This should consist of infinitely many distinct intervals. Then show why, collectively, the intervals contain infinitely many integers.

2577. Consider the single transformation which maps one parabola onto the other.

2578. (a) Draw a tree diagram, conditioned on it raining or not.
(b) Restrict the possibility space to two branches.

2579. (a) $x = y^2$ is a parabola,
(b) Use definite integration with respect to y.

2580. (a) Draw a force diagram. Resolve vertically.
(b) Resolve horizontally.
(c) No calculation is needed here.

2581. (a) Apply the iteration twice to x, and equate the result to x.
(b) Spot two easy roots, then solve the remaining quadratic. Consider whether all of these four are answers to the problem.

2582. You don't need to find θ, indeed it's easier if you don't. Square both equations, and then subtract them to form one side of a Pythagorean identity.

2583. (a) This is a one-tailed test.
(b) Find the critical region, or a p-value.
(c) Consider what happens if the same sample is used to generate a suspicion and then also to test it.

2584. This needs a careful sketch. The reason for there being an *inequality*, as opposed to an *equation*, is the fact that a quadrilateral is not fixed in shape from its four side lengths. So, to show this result, you need to calculate $\arccos \frac{5}{6}$ as the angle in the most extreme version of the quadrilateral.

2585. Sketch the boundary equations, and find, in terms of a and b, the coordinates of any intersections. Then set up a single definite integral.

2586. Set the first derivative to zero, and so express the x coordinates of any SPs as $x = p \pm q$. Then set the second derivative to zero to find the point of inflection.

2587. Both equations can be factorised.

2588. Use the fact that the curves are reflections in $y = x$, together with the fact that the shortest distance is along the normal.

2589. (a) Consider the fact that the distributions within the range are very different.
(b) Look for the greatest density of marks.

2590. Find a counterexample: two different functions for which the order of application makes no difference.

2591. Think graphically.

2592. (a) Substitute for y, and set $x = a$. Take out a common factor of $\sqrt{x}$ and rearrange.
(b) The RHS from (a) must be well defined.

2593. Use log rules to write the LHS as a single logarithm. Then exponentiate both sides, over base 2. Then rearrange and take natural logarithms.

2594. The centre of the rotational symmetry of a cubic is its point of inflection.

2595. None are directly proportional, but one pair has a linear relationship. Integrate the given equation to find it.

2596. Rearrange and cube root both sides. Then convert sin and cos into tan.

2597. Use the chain rule to find the second derivative y''. Substitute into the LHS as an expression, then simplify to get the RHS.

2598. The statement is incorrect. Consider NIII.

2599. The boundary equation describes a sphere.

2600. (a) Differentiate wrt x and then reciprocate.
(b) This is an instance of the chain rule.
(c) Put parts (a) and (b) together.

—— End of 26th Hundred ——